## Long-Run Production and Costs

In the previous section we learned the details of firm production and the costs of that production in the short-run. In this section we will discuss almost the same concepts - the firm's production and costs of production - with the only difference being that now we consider these in the long-run instead of the short-run. The first part of this chapter discusses production in the long-run while in the second part we discuss how production affects costs and what those costs consist of in the long-run.

## I. Production in the Long-Run

Recall that the long-run is a period of time during which all productive resources can be changed by the firm (but technology remains fixed). In contrast, in the short-run some (at least one) productive resource cannot be changed by the firm. Thus, the difference between the short-run and the long-run is whether or not the firm has complete control over its resources and can vary them at will; in the long-run firms face more choices because they can more completely control their resources.

Next, let us turn to the graphical representation of production in the long-run. Obviously, this will be more complex exactly because of the additional choices the firm must make regarding all of its resource levels rather than just a few. Consider again the short-run graphs we presented in the chapter on short-run production and costs.
In the simple example we were using in the previous chapter the firm had two resources, labor and capital, with labor variable and capital fixed. Graph 1 shows how labor (L) affects output with capital (K) fixed. How do we graphically add even one more resource to Graph 1?

Two methods will be used to show a firm's production function in the long-run. First, Graph 2 shows how capital affects output by shifting the short-run production function as the amount of capital available changes. This illustration of long-run production will again use the example of teenagers (labor) using shovels (capital) to clean out irrigation ditches.
Graph 2 shows that with the same amount of labor, ten teenagers, that output rises as the amount of capital utilized by the firm increases. For example, with five shovels, total output is 15 yards cleared per day. When another teenager is hired, output rises to 22 yards and when the seventh teenager is hired, output rises to 25 yards.
Graph 2 illustrates three separate short-run production functions, one for each of the three different levels of shovels. Notice that each of these production functions has the same

properties that we discussed in the previous chapter. That is, in each case diminishing returns apply to the increased use of labor, with capital fixed, in the short-run.

Does the concept of diminishing returns also apply to the firm's use of capital, holding labor fixed? That is, if we hold the number of teenagers fixed and increase the number of shovels will the increase in output for each successive extra shovel eventually decrease? Notice that what is really being asked here is whether or not the marginal product of capital also must eventually decline. The marginal and average product of capital is defined similarly as for labor:

$$
\begin{equation*}
\text { Marginal Product of Capital }\left(\mathrm{MP}_{\mathrm{K}}\right)=\frac{\Delta Q}{\Delta K} \tag{1}
\end{equation*}
$$

$$
\begin{equation*}
\text { Average Product of Capital }\left(\mathrm{MP}_{\mathrm{K}}\right)=\frac{Q}{K} \tag{2}
\end{equation*}
$$

In Graph 2, the $\mathrm{MP}_{\mathrm{K}}$ of the sixth shovel equals 7 while the $\mathrm{MP}_{\mathrm{K}}$ of the seventh shovel equals 3 . Notice that in Graph 2 capital also has diminishing returns. The second method that will be used to add capital in the long-run looks at the relationship between capital and output. Notice that because the law of diminishing returns also applies to capital that the relationship between capital and output shown in Graph 3 also has diminishing return.

Because diminishing returns also applies to capital the average and marginal product of capital must look similar to those for labor. As shown in Graph 4, both have an inverted $U$ shape, with $\mathrm{MP}_{\mathrm{K}}$ first rising and then falling as more capital is utilized.


## II. Least Cost Production in the Long-Run

A firm actually has a more difficult and complex series of decisions in the long-run than in the short-run. Again, consider our simple production process with only two inputs, teenagers (labor) and shovels (capital). In the short-run, the firm must decide how much output to produce to maximize their profit. Once the output level is decided, then the production function informs the firm how many workers must be hired. For example, suppose the firm has five shovels and wishes to clear 15 yards of ditch per day. According to the production function in Graph 2 with five shovels, the firm must hire five workers.

However, in the long-run a firm can change both labor and capital, which means that more than one method of achieving a given level of output must exist. For example, Graph 5 illustrates three different methods of producing an output of 15 : (1) L=5, K=5 (2) $\mathrm{L}=4, \mathrm{~K}=6$ and (3) $\mathrm{L}=3, \mathrm{~K}=7$. Given that all of these methods produce the same output, which should the firm choose to use?

What is the firm's goal? Recall that we are assuming that the firm wants to maximize its profits. Recall that a firm's profits equal its total revenue less its total costs. Therefore, the firm has two different methods by which it can increase its profits; increase its total revenue or reduce its total costs.


Will the three different methods of producing
15 units of output yield different total costs? To be able to calculate total costs one must first know the price of labor ( w ) and capital $\left(\mathrm{p}_{\mathrm{K}}\right)$. Suppose that the firm is paying fifty dollars a day for each worker and twenty dollars for each shovel. Now calculate total costs for each of the three methods of producing 15 units of output.

Method 1: Total Cost $=w^{*} L+p_{k} * K=50 * 5+20 * 5=250+100=\$ 350$
Method 2: Total Cost $=w^{*} L+p_{k}{ }^{*} K=50 * 4+20 * 6=200+120=\$ 320$
Method 3: Total Cost $=w^{*} \mathrm{~L}+\mathrm{p}_{\mathrm{k}}{ }^{*} \mathrm{~K}=50 * 3+20 * 7=150+140=\$ 290$
Now it becomes clearer which of these three methods will the firm prefer to use. First note that the firm's total revenue is the same regardless of the method used because in each the firm produces the same level of output. Hence, a profit maximizing firm would choose method 3 because costs of production are lower. We can conclude, therefore, that:

A profit maximizing firm will choose its inputs in the long-run so as to minimize its costs of production.

What would cause a firm to use more, or less, of an input? Clearly, one of the variables that must matter is the price of the input. In the example given above the price of labor is $\$ 50$ while the price of capital is only $\$ 20$ and, as a result, the firm chooses the least labor intensive production method. How does the result change if the prices change? For example, suppose that the prices were reversed with wages of $\$ 20$ and the price of capital of $\$ 50$. If you calculate the total costs for each production method now you will find that the most labor intensive method has the lowest total costs.

Another variable that must matter in the decision is the marginal productivity of each input. For example, even if capital were quite costly compared to labor the firm might choose to
nonetheless utilize a lot of capital if it were also very productive. Similar to our discussion of maximizing utility it turns out that a firm will minimize the costs of producing a given level of output by choosing inputs such that:

$$
\begin{equation*}
\frac{\mathrm{MP}_{\mathrm{L}}}{\mathrm{w}}=\frac{\mathrm{MP}_{\mathrm{K}}}{\mathrm{p}_{\mathrm{K}}} \tag{3}
\end{equation*}
$$

Obviously, if the firm had more than just two inputs being used in the production process, then the same general rule would apply; the level of each input should be chosen such that the ratio of each input's marginal product to its price are equal.
Why would using inputs so these ratios are equal lead to the optimal, cost-minimizing input usage? To understand, you must first understand the meaning of the ratio itself. Suppose that the $M P_{\llcorner }$is 100 while the wage is $\$ 50$. The ratio in equation $3, M P_{\llcorner } / w$, equals two. That is, for each of the 50 dollars being spent on an extra worker the firm gains two more units of production. Thus, the marginal product of an input divided by its price represents the extra output gained by the firm by spending one more dollar on that input. Clearly, the firm would want to spend its money first on inputs with the highest return per dollar. Thus, suppose that the two ratios are as given in equation 4 below.

$$
\begin{equation*}
\frac{100}{50}=2=\frac{\mathrm{MP}_{\mathrm{L}}}{\mathrm{w}}>\frac{\mathrm{MP}_{\mathrm{K}}}{\mathrm{p}_{\mathrm{K}}}=\frac{20}{20}=1 \tag{4}
\end{equation*}
$$

In this case, an extra dollar spent on labor will yield 2 units of output while an extra dollar spent on capital will yield 1 unit of output. Clearly in this case a profit maximizing - cost minimizing - firm would increase its use of labor and decrease its use of capital. Notice that increasing labor decreases $\mathrm{MP}_{\mathrm{L}}$ (how do we know this?) while decreasing capital increases the $\mathrm{MP}_{\mathrm{K}}$, moving equation four towards equality. ${ }^{1}$ The firm will continue to increase labor and decrease capital as long as the two ratios are unequal as they are in equation 4; until the two ratios are equal and the firm is minimizing its costs of production.
Table 1 illustrates how the firm responds when its current input levels results in those inputs having unequal marginal product per dollar.

[^0]| Table 1 <br> Firm Response to Unequal $M P_{L} / w$ and $M P_{K} / P_{K}$ |  |
| :---: | :---: |
| $\frac{\mathrm{MP}_{\mathrm{L}}}{\mathrm{w}}>\frac{\mathrm{MP}_{\mathrm{K}}}{\mathrm{p}_{\mathrm{K}}}$ | $\frac{\mathrm{MP}_{\mathrm{L}}}{\mathrm{w}}<\frac{\mathrm{MP}_{\mathrm{K}}}{\mathrm{p}_{\mathrm{K}}}$ |
| $\uparrow L$ and $\downarrow \mathrm{K}$ | $\uparrow \mathrm{K}$ and $\downarrow \mathrm{L}$ |
| $\mathrm{MP} \mathrm{L}_{\mathrm{L}} \downarrow$ and $\mathrm{MP}_{\mathrm{K}} \uparrow$ | $\mathrm{MP} \mathrm{L} \uparrow$ and $\mathrm{MP}_{\mathrm{K}} \downarrow$ |
| In both cases, the action continues until $\frac{\mathrm{MP}_{\mathrm{L}}}{\mathrm{w}}=\frac{\mathrm{MP}_{\mathrm{K}}}{\mathrm{p}_{\mathrm{K}}}$ |  |

Make sure that you understand and can replicate the results of Table 1. Suppose that a firm is currently choosing its inputs in a way that minimizes its costs of production and one of the variables changes. For example, suppose that wages increase. How will the firm respond to this change? Table 2 illustrates the variables that can change and how the firm will respond to these changes according to the principles that we've just developed.

## Table 2

Firm Response to Changes in Underlying Variables

$$
\text { Initially } \frac{\mathrm{MP}_{\mathrm{L}}}{\mathrm{w}}=\frac{\mathrm{MP}_{\mathrm{K}}}{\mathrm{p}_{\mathrm{K}}} \text { in all cases }
$$

=> The firm is choosing its inputs to minimize its production costs.

| Now $w \uparrow=>$ | Now $P_{K} \uparrow=>$ | Now $M P_{L} \uparrow=>$ | Now $M P_{K} \uparrow=>$ |
| :---: | :---: | :---: | :---: |
| $\frac{M P_{L}}{w}<\frac{M P_{K}}{p_{K}}$ | $\frac{M P_{L}}{w}>\frac{M P_{K}}{p_{K}}$ | $\frac{M P_{L}}{w}>\frac{M P_{K}}{p_{K}}$ | $\frac{M P_{L}}{w}<\frac{M P_{K}}{p_{K}}$ |
| The Firm will respond by: |  |  |  |
|  |  |  |  |

For each column, the reverse actions will occur at each step if the variable changes in the opposite manner.

## III. Costs in the Long-Run

Recall that production was more complicated in the long-run because the firm has more choices. In contrast, though, production costs are actually much less complicated in the long-run. In the long-run all inputs can be changed. This means that there are no fixed costs or all costs are variable. Hence, in the long-run we no longer need to make the distinction between fixed and variable costs.
A. Long-Run Average Cost Defined

To further simplify the discussion of long-run costs, we will only consider average costs in the long-run or long-run average costs (LRAC). Graph 6 will be used to help define LRAC. The graph shows three different points; three different levels of average costs in the long-run the firm faces to produce an output of 100 units. These three points actually represent three different types of average costs. Points like A are not attainable. That is, with current technology, production methods and input prices it is not possible to produce an output of 100 for an average cost of $\$ 7$.

Points like B and C are both attainable with the difference being that point $B$ represents the minimum average cost possible. That is, with current technology, production methods and input prices the minimum average cost possible to produce 100 units of output is \$15.


Thus, at point $B$ the firm is choosing its
Graph 6 inputs to minimize its costs of production as discussed in section II above. Obviously, while the firm cannot produce 100 units at point $A$ it can produce 100 units at point $C$ by simply choosing a production method that does not minimize costs.

Point $B$ is on the long-run average cost curve. In fact, the long-run average cost curve consists of all the points like B for each different level of output. Put another way, the long-run average cost curve shows the minimum average cost of producing any given level of output.
B. The Long-Run Average Cost Curve Graphically

What does the long-run average cost curve look like? Remember that average costs in the short-run were u-shaped because of: (1) increasing returns initially due to specialization of labor and (2) decreasing returns eventually due to congestion. Consider whether either or both of these apply to average costs in the long-run keeping in mind the differences between production in the short and long-run.

First, as production initially increases will there be increasing returns, causing LRAC to decrease due to specialization of labor? Clearly, the answer here is yes. There exists no substantive difference in the short-run and the long-run for specialization of labor. Increasing output and hiring more labor does mean in both the short and the long-run that the labor can become more specialized and, in turn, causing productivity to rise and costs to fall. Further, in the long-run because capital can also be increased then it can also become more specialized, again, causing productivity to rise and costs to fall.

Second, does the law of diminishing returns also apply in the long-run? Recall that diminishing returns in the short-run results from increasing one input while holding the other input constant. In our example above, we increased the number of teenagers while holding the number of shovels constant. Eventually, congestion resulted and productivity decreased, increasing average costs. In the long-run, though, no such congestion occurs because both teenagers and shovels can be increased simultaneously.

However, something similar to congestion does occur in the long-run that will also eventually cause diminishing returns.
Consider a firm that is increasing its output in the long-run by increasing all of its inputs. In essence, the firm is simply getting larger and larger. The firm must also spend more resources in controlling the firm's operation. Controlling the firm's operations includes simply directing different parts of the firm in what they should produce and when. Controlling the firm also includes reducing shirking by workers. Eventually, control of the firm becomes so difficult given the current technology that diminishing returns sets in and long-run average costs will begin to rise.


Graph 7 shows the u-shaped long-run
Graph 7 average cost curve. Notice that point $B$ is on the cost curve while point $A$ is below it and point $C$ above it.
C. Definitions related to LRAC

The final step in our discussion of long-run costs is to more completely define a number of terms relating to the long-run average cost curve. First, consider what is meant by economists when discussing the "scale" of a firm's operations. Essentially, as a firm increases its output in the long-run it does so by increasing all of its inputs, as noted above, and becoming larger; this is referred to as increasing scale of operations. Note that if a firm increased its output in the short-run it would not be increasing its scale because the firm is not increasing all of its inputs. Thus, scale is only a long-run concept.
Graph 8 illustrates the definitions of four additional long-run average cost concepts. To understand each of these concepts you must understand that in the long-run when a
 firm increases the quantity of output it produces it is also increasing its scale or size of operations.

1. Increasing Returns to Scale or Economies of Scale.

Recall that specialization of labor and other inputs causes a firm to have increasing returns in the long-run as output (scale) increases. However, increasing returns as output (scale) increases causes the LRAC to decrease. Thus, as shown by Graph 8, increasing returns to scale or economies of scale occurs when LRAC decreases as output (scale) increases.
2. Constant Returns to Scale

It is quite common for industries to have portions of their LRAC that is flat bottomed. This can only happen if, as scale increases, input productivity and LRAC remains constant. As Graph 8 shows, constant returns to scale occurs when the LRAC does not change as output (scale) increases.
3. Decreasing Returns to Scale or Diseconomies of Scale.

Recall that even in the long-run eventually as scale increases the firm will experience increasing difficulty in controlling the firm and motivating its workers, causing diminishing returns and increasing LRAC as output (scale) increases. Thus, as Graph 8 shows, decreasing returns to scale or diseconomies of scale occur when LRAC increases as output (scale) increases.
4. Minimum Efficient Scale.

Finally, consider the size or scale a firm in a given industry must achieve to be technologically efficient. Recall that technological efficiency requires that a firm produce its output at the minimum cost possible. In Graph 8, what scale or level of output would be required for a firm to produce at the lowest LRAC? In other words, how large does the firm have to become in order to take advantage of all economies of scale?

Clearly firms must be producing in the flat bottomed portion of Graph 8 in order to achieve this type of technological efficiency. However, the concept of minimum efficient scale only requires a firm to achieve the lowest output (scale) that is required to achieve efficiency. Graph 8 illustrates that this occurs where the LRAC first reaches its minimum point.


[^0]:    ${ }^{1}$ The law of diminishing returns implies that as a firm uses more or less of an input its marginal product must also decrease or increase, respectively.

