MTH 261 Algebra and Trig Review

A. Algebra Review

1. Evaluate each expression without using a calculator
   (a) \((-3)^4\) \hspace{2cm} (b) \(-3^4\) \hspace{2cm} (c) \(3^{-4}\)
   (d) \(\frac{5^{23}}{5^{21}}\) \hspace{2cm} (e) \(\left(\frac{2}{3}\right)^{-2}\) \hspace{2cm} (f) \(16^{-3/4}\)

2. Simplify each expression. Write your answer without negative exponents.
   (a) \(\sqrt{200} - \sqrt{32}\)
   (b) \((3a^3b^3)(4ab^2)^2\)
   (c) \(\left(\frac{3x^{3/2}y^3}{x^{2}y^{-1/2}}\right)^{-2}\)

3. Expand and simplify.
   (a) \(3(x + 6) + 4(2x - 5)\) \hspace{2cm} (b) \((x + 3)(4x - 5)\)
   (c) \((\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b})\)
   (d) \((2x + 3)^2\)
   (e) \((x + 2)^3\)

4. Factor each expression.
   (a) \(4x^2 - 25\)
   (b) \(2x^2 + 5x - 12\)
   (c) \(x^3 - 3x^2 - 4x + 12\)
   (d) \(x^4 + 27x\)
   (e) \(3x^{3/2} - 9x^{1/2} + 6x^{-1/2}\)
   (f) \(x^3y - 4xy\)

5. Simplify the rational expression.
   (a) \(\frac{x^2 + 3x + 2}{x^2 - x - 2}\)
   (b) \(\frac{2x^2 - x - 1}{x^2 - 9} \cdot \frac{x + 3}{2x + 1}\)
   (c) \(\frac{x^2}{x^2 - 4} - \frac{x + 1}{x + 2}\)
   (d) \(\frac{\frac{y}{z} - \frac{x}{z}}{\frac{y}{z} - \frac{1}{z}}\)

6. Rationalize the expression and simplify.
   (a) \(\frac{\sqrt{19}}{\sqrt{5} - 2}\)
   (b) \(\frac{\sqrt{4 + h} - 2}{h}\)

7. Rewrite by completing the square.
   (a) \(x^2 + x + 1\)
   (b) \(2x^2 - 12x + 11\)
8. Solve the equation. (Find only the real solutions.)

(a) \( x + 5 = 14 - \frac{1}{2}x \)  
(b) \( \frac{2x}{x + 1} = \frac{2x - 1}{x} \)
(c) \( x^2 - x - 12 = 0 \)  
(d) \( 2x^2 + 4x + 1 = 0 \)
(e) \( x^4 - 3x^2 + 2 = 0 \)  
(f) \( 3|x - 4| = 10 \)
(g) \( 2x(4 - x)^{-1/2} - 3\sqrt{4-x} = 0 \)

9. Solve each inequality. Write your answer using interval notation.

(a) \( -4 < 5 - 3x \leq 17 \)  
(b) \( x^2 < 2x + 8 \)
(c) \( x(x - 1)(x + 2) > 0 \)  
(d) \( |x - 4| < 3 \)
(e) \( \frac{2x - 3}{x + 1} \leq 1 \)

10. State whether each equation is true or false.

(a) \( (p + q)^2 = p^2 + q^2 \)  
(b) \( \sqrt{ab} = \sqrt{a}\sqrt{b} \)
(c) \( \sqrt{a^2 + b^2} = a + b \)  
(d) \( \frac{1 + TC}{C} = 1 + T \)
(e) \( \frac{1}{x - y} = \frac{1}{x} - \frac{1}{y} \)  
(f) \( \frac{1/x}{a/x - b/x} = \frac{1}{a - b} \)

B. Analytic Geometry Review

1. Find an equation for the line that passes through the point \((2, -5)\) and

(a) has slope \(-3\)
(b) is parallel to the \(x\)-axis
(c) is parallel to the \(y\)-axis
(d) is parallel to the line \(2x - 4y = 3\)

2. Find an equation for the circle that has center \((-1, 4)\) and passes through the point \((3, -2)\).

3. Find the center and radius of the circle with equation \(x^2 + y^2 - 6x + 10y + 9 = 0\).

4. Let \(A(-7, 4)\) and \(B(5, -12)\) be points in the plane.

(a) Find the slope of the line that contains \(A\) and \(B\).
(b) Find an equation of the line that passes through \(A\) and \(B\). What are the intercepts?
(c) Find the midpoint of the segment \(AB\).
(d) Find the length of the segment \(AB\).
(e) Find an equation of the perpendicular bisector of \(AB\).
(f) Find an equation of the circle for which \(AB\) is a diameter.
5. Sketch the region in the $xy$-plane defined by the equation of inequalities.
   (a) $-1 \leq y \leq 3$  
   (b) $|x| < 4$ and $|y| < 2$
   (c) $y < 1 - \frac{1}{2}x$
   (d) $y \geq x^2 - 1$
   (e) $x^2 + y^2 < 4$
   (f) $9x^2 + 16y^2 = 144$

C. Functions Review

1. The graph of a function $f$ is shown.
   (a) State the value of $f(-1)$.
   (b) Estimate the value of $f(2)$.
   (c) For what values of $x$ is $f(x) = 2$?
   (d) Estimate the values of $x$ such that $f(x) = 0$.
   (e) State the domain and range of $f$.

2. If $f(x) = x^3$, evaluate the difference quotient $\frac{f(2 + h) - f(2)}{h}$ and simplify your answer.

3. Find the domain of the function.
   (a) $f(x) = \frac{2x + 1}{x^2 + x - 2}$
   (b) $g(x) = \frac{\sqrt{x}}{x^2 + 1}$
   (c) $h(x) = \sqrt{4 - x + \sqrt{x^2 - 1}}$

4. How are the graphs of the functions obtained from the graph of $f$?
   (a) $y = -f(x)$
   (b) $y = 2f(x) - 1$
   (c) $y = f(x - 3) + 2$

5. Without using a calculator, make a rough sketch of the graph.
   (a) $y = x^3$
   (b) $y = (x + 1)^3$
   (c) $y = (x - 2)^3 + 3$
   (d) $y = 4 - x^2$
   (e) $y = \sqrt{x}$
   (f) $y = 2\sqrt{x}$
   (g) $y = -2x$
   (h) $y = 1 + x^{-1}$

6. Let $g(x) = \begin{cases} 
1 - x^2 & \text{if } x \leq 0 \\
2x + 1 & \text{if } x > 0 
\end{cases}$
   (a) Evaluate $f(-3)$ and $f(1)$.
   (b) Sketch the graph of $f$.

7. If $f(x) = x^2 + 2x - 1$ and $g(x) = 2x - 3$, find each of the following functions
   (a) $f \circ g$
   (b) $g \circ f$
   (c) $g \circ g \circ g$
D. Trigonometry Review

1. Convert from degrees to radians.
   (a) $300^\circ$  
   (b) $-18^\circ$

2. Convert from radians to degrees.
   (a) $\frac{5\pi}{6}$  
   (b) $2$

3. Find the length of an arc of a circle with radius 12 cm if the arc subtends a central angle of $30^\circ$.

4. Find the exact values.
   (a) $\tan\left(\frac{\pi}{3}\right)$  
   (b) $\sin\left(\frac{7\pi}{6}\right)$  
   (c) $\sec\left(\frac{5\pi}{3}\right)$

5. In a right triangle the side opposite angle $\theta$ has length $a$, the side adjacent to $\theta$ has length $b$, and the hypotenuse has length 24. Express $a$ and $b$ in terms of $\theta$.

6. If $\sin x = \frac{1}{3}$ and $\sec y = \frac{5}{4}$, where $x$ and $y$ lie between 0 and $\pi/2$, evaluate $\sin(x + y)$.

7. Prove the identities.
   (a) $\tan \theta \sin \theta + \cos \theta = \sec \theta$
   (b) $\frac{2 \tan x}{1 + \tan^2 x} = \sin 2x$

8. Find all values of $x$ such that $\sin 2x = \sin x$ and $0 \leq x \leq 2\pi$.

9. What is the period of $\sin x$, $\cos x$, and $\tan x$?

10. Without using a calculator, sketch one period of $\sin x$, $\cos x$, $\tan x$, and $\sec x$.

11. Sketch the graph of the function $y = 1 + \sin 2x$ without using a calculator.